



Properties of DTFS

Q2 Find the DTFS and sketch the amp and phase spectra.

Q2 $x(n) = 5 + \sin\left(\frac{n\pi}{2}\right) + \cos\left(\frac{n\pi}{4}\right)$

$N_1 = \frac{2\pi}{\omega_0} m = \left(\frac{2\pi}{\pi/2}\right) \times m = 4$ for $m=1$

$N_2 = \left(\frac{2\pi}{\omega_0}\right) \times m = 8$ $\frac{N_1}{N_2} = \frac{4}{8} = \frac{1}{2}$

$2N_1 = N_2 = 8$

$x(n) = 5 + \sin\left(\frac{n\pi}{2}\right) + \cos\frac{n\pi}{4}$
 $= 5 + \left[\frac{e^{j\frac{n\pi}{2}} - e^{-j\frac{n\pi}{2}}}{2j} \right] + \frac{e^{j\frac{n\pi}{4}} + e^{-j\frac{n\pi}{4}}}{2}$

$x(n) = \sum_{k=-3}^4 C_k e^{j2\pi kn/N}$ $N=8$

on comparing eqn 1 and eqn 2

$= C_{-3} e^{-j3\pi n/4} + C_{-2} e^{-j\pi n/2} + C_{-1} e^{-j\pi n/4} + C_0 + C_1 e^{j\pi n/4}$
 $+ C_2 e^{j\pi n/2} + C_3 e^{j3\pi n/4} + C_4 e^{j\pi n}$

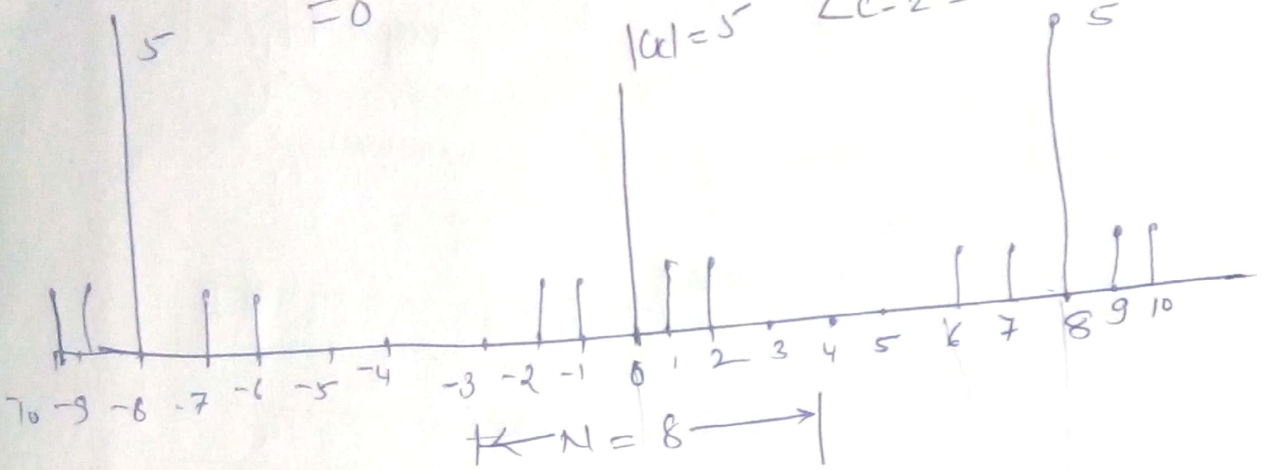
$C_{-2} = -\frac{1}{2j} = \frac{j}{2}$ $C_2 = \frac{1}{2j}$
 $C_0 = 5$; $C_1 = \frac{1}{2}$; $C_{-1} = \frac{1}{2}$

$|C_k| = \frac{1}{2}$, $k = -1, -2, 1, 2$ $\sqrt{0.5^2 + 0.5^2} = 0.5$
 $= 5$, $k=0$
 $= 0$, $k = -3, 3, 4, -4$



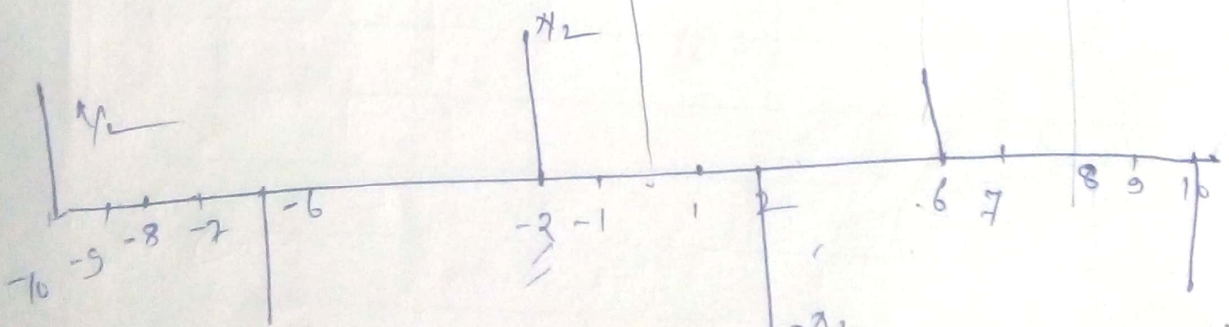


$\angle C_k = \frac{\pi}{2}$, for $k = 2$
 $k-2 = \frac{-1}{2j} = \frac{-j}{2} = 0.5j$
 $\angle C_{-2} = \pi/2$, $k = -2$ $\angle C_{-2} = \tan^{-1} \left(\frac{0.5}{0} \right) = \infty = \pi/2$
 $= \pi/2$ for $k = 2$ $|C_2| = \frac{1}{2j} = \frac{j}{2} = -0.5j$
 $= 0$ $|C_2| = \frac{1}{2j} = \frac{j}{2} = -0.5j$
 $\angle C_{-2} = \pi/2$



Magnitude spec.

Phase spectrum



Mag
 $|C_k|$

coefficients

$C_2 = \frac{1}{2j}$
 $|C_2| = \frac{1}{2j} = \frac{1 \times j}{2j \times j} = \frac{j}{2(-1)} = -0.5j$
 $= 0 - 0.5j = \sqrt{0^2 + 0.5^2} = 0.5$

$|C_2| = \frac{-1}{2j} = 0 + 0.5j = 0.5$
 $|C_1| = \frac{1}{2}$ $|C_{-1}| = \frac{1}{2}$

$\angle = \tan^{-1} \left(\frac{0.5}{0} \right) = \tan^{-1}(\infty) = \pi/2$
 $\angle C_{-2} = \tan^{-1}(\pi/2)$
 Real term
 $\angle C_1 = \frac{0.5}{0.5} = \tan^{-1}(1)$
 $\angle C_{-1} = 0$





Q.3 $x(n) = \cos\left(\frac{6\pi n}{13} + \frac{\pi}{6}\right)$

$N = \frac{2\pi \times 13}{6\pi \times 3} \times m$

$= m=3, N=13$ samples

Synthesizer $x(n) = \sum_{k=-N}^N x(k) e^{jk\omega_0 n}$

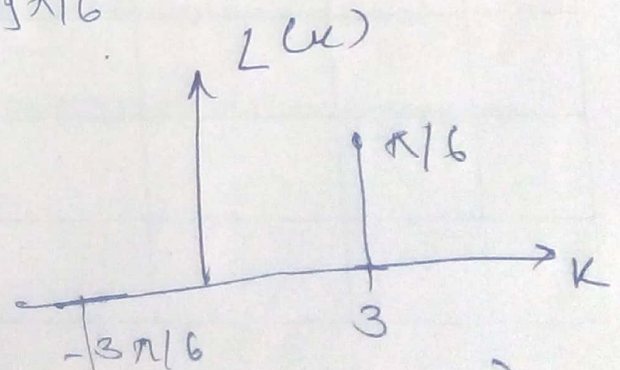
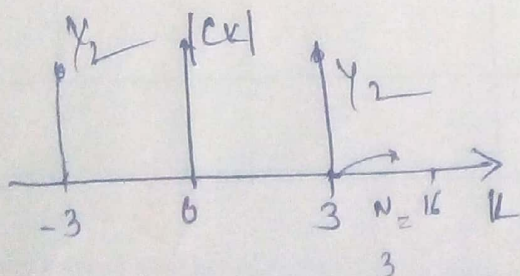
given eqn = $\frac{e^{j\left(\frac{6\pi}{13}n + \frac{\pi}{6}\right)} + e^{-j\left(\frac{6\pi}{13}n + \frac{\pi}{6}\right)}}{2}$

$= \frac{1}{2} e^{j\frac{6\pi}{13}n} e^{j\frac{\pi}{6}} + \frac{1}{2} e^{-j\frac{6\pi}{13}n} e^{-j\frac{\pi}{6}}$
 $= \frac{1}{2} e^{j\frac{\pi}{6}} e^{j \times 3 \left(\frac{2\pi}{13}\right)n} + \frac{1}{2} e^{-j\frac{\pi}{6}} e^{-j \times 3 \left(\frac{2\pi}{13}\right)n}$
 $k=3$

$x(n) = \sum_{k=-N}^N C(k) e^{jk\omega_0 n}$

$X(3) = \frac{1}{2} e^{j\frac{\pi}{6}} \rightarrow \theta = \frac{\pi}{6}$

$X(-3) = \frac{1}{2} e^{-j\frac{\pi}{6}} \rightarrow \theta = -\frac{\pi}{6}$



$X(3) = \frac{1}{2} \left(\cos \frac{\pi}{6} + j \sin \frac{\pi}{6} \right)$

$X(-3) = \frac{1}{2} \left(\cos \frac{\pi}{6} - j \sin \frac{\pi}{6} \right)$

$|X_3| = \sqrt{\left(\frac{1}{2}\right)^2 \cos^2 \frac{\pi}{6} + \left(\frac{1}{2}\right)^2 \sin^2 \frac{\pi}{6}} = \frac{1}{2}$
 or $\tan^{-1} \left(\frac{1/2 \sin \pi/6}{1/2 \cos \pi/6} \right)$

